The two complex conjugate roots of $\sqrt[3]{-8}$ may be verified by simple expansion as shown below.

$$
\begin{aligned}
(1 + i\sqrt{3})^3 &= (1 + i\sqrt{3})(1 + i\sqrt{3})(1 + i\sqrt{3}) && \text{expand first two factors} \\
&= \left[1 + i\sqrt{3} + i\sqrt{3} + (i)^2(\sqrt{3})^2\right]\left[(1 + i\sqrt{3}\right] && i^2 = -1 \\
&= (1 + 2i\sqrt{3} - 3)(1 + i\sqrt{3}) \\
&= (2i\sqrt{3} - 2)(1 + i\sqrt{3}) && \text{expand these two factors} \\
&= 2(i\sqrt{3} - 1)(1 + i\sqrt{3}) && \text{actually this is a difference of two squares} \\
&= 2(i\sqrt{3} + (i\sqrt{3})^2 - 1 - i\sqrt{3}) \\
&= 2\left[-3 - 1\right] \\
&= -8.
\end{aligned}
$$

$$
\begin{aligned}
(1 - i\sqrt{3})^3 &= (1 - i\sqrt{3})(1 - i\sqrt{3})(1 - i\sqrt{3}) && \text{expand first two factors} \\
&= \left[1 - i\sqrt{3} - i\sqrt{3} + (+i)^2(\sqrt{3})^2\right]\left[(1 - i\sqrt{3}\right] && i^2 = -1 \\
&= (1 - 2i\sqrt{3} - 3)(1 - i\sqrt{3}) \\
&= (-2i\sqrt{3} - 2)(1 - i\sqrt{3}) && \text{expand these two factors} \\
&= -2(i\sqrt{3} + 1)(1 - i\sqrt{3}) && \text{actually this is a difference of two squares} \\
&= -2(i\sqrt{3} - (i\sqrt{3})^2 + 1 - i\sqrt{3}) \\
&= -2\left[-(i^2)(3) + 1\right] \\
&= -2\left[3 + 1\right] \\
&= -8.
\end{aligned}
$$

Instead of this tedium, it is easier and smarter to use polar coordinates and get the result faster, and with less scope for error. Our root verification then becomes:

$$
\left[2e^{\frac{i\pi}{3}}\right]^3 = 8e^{i\pi} = 8(-1) = -8.
$$

$$
\left[2e^{\frac{i5\pi}{3}}\right]^3 = 8e^{i5\pi} = 8e^{-i\pi} = 8(-1) = -8.
$$

This should impress you enough that you should use polar representations for multiplication and Cartesian representations for addition.